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Properties of Q-Intuitionistic L-Fuzzy Subsemigroups of A Semigroup

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Abstract: In this paper, we study the properties of Q-intuitionistic L-fuzzy subsemigroup of a semigroup and prove some results on these.
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I. INTRODUCTION

After the introdution of fuzzy sets by L.A.Zadeh[15], several researchers explored on the generalization of the notion of fuzzy set. The concept of intuitionistic L-fuzzy subset was introduced by K.T.Atanassov[4,5], as a generalization of the notion of fuzzy set. Azriel Rosenfeld[6] defined a fuzzy groups. Asok Kumer Ray[3] defined a product of fuzzy subgroups and A.Solairaju and R.Nagarajan[13,14] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. We introduce the concept of Q-intuitionistic L-fuzzy subsemigroup of a semigroup and established some results.

1.PRELIMINARIES:

1.1 Definition: Let X be a non-empty set and L = (L, $\leq$) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A **(Q, L)-fuzzy subset** A of X is a function A : XxQ $\to$ L.

1.2 Definition: Let (L, $\leq$) be a complete lattice with an involutive order reversing operation N : L $\to$ L and Q be a non-empty set. A **Q-intuitionistic L-fuzzy subset** (QILFS) A in X is defined as an object of the form A={< (x, q), μ_A(x, q), ν_A(x, q) > / x in X and q in Q }, where μ_A : XxQ $\to$ L and ν_A : XxQ $\to$ L define the degree of membership and the degree of non-membership of the element x$\in$X respectively and for every x$\in$X satisfying $\mu_A(x) \leq N(\nu_A(x))$.

1.3 Definition: Let (R, .) be a semigroup. A Q-intuitionistic L-fuzzy subset A of R is said to be a Q-intuitionistic L-fuzzy subsemigroup(QILFSSG) of R if it satisfies the following axioms:

(i) $\mu_A(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$
(ii) $\nu_A(xy, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$, for all x and y in R and q in Q.

1.4 Definition: Let X and X′ be any two sets. Let f : X $\to$ X′ be any function and A be a Q-intuitionistic L-fuzzy subset in X, V be a Q-intuitionistic L-fuzzy subset in f(X) = X′, defined by $\mu_V(y, q) = \sup_{x\in f^{-1}(y)} \mu_A(x, q)$ and $\nu_V(y, q) = \inf_{x\in f^{-1}(y)} \nu_A(x, q)$, for all x in X and y in X′. A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.5 Definition**:** Let A be a Q-intuitionistic L-fuzzy subsemigroup of a semigroup (R, $\cdot$) and a in R. Then the **pseudo** Q-intuitionistic **L-fuzzy coset** $(aA)^p$ is defined by $((a\mu_A)^p)(x, q) = p(a)\,\mu_A(x, q)$ and $((a\nu_A)^p)(x, q) = p(a)\nu_A(x, q)$, for every x in R and for some p in P and q in Q.

II. SOME PROPERTIES OF Q-INTUITIONISTIC L-FUZZY SUBSEMIGROUPS OF A SEMIGROUP

2.1 Theorem: If A is a Q-intuitionistic L-fuzzy subsemigroup of a semigroup (R, $\cdot$), then H = { x / x$\in$R : μ_A(x, q) = 1, ν_A(x, q) = 0} is either empty or is a subsemigroup of R.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $\mu_A(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q) = 1\wedge 1= 1$. Therefore, $\mu_A(xy, q) = 1$. And $\nu_A(xy, q) \leq \nu_A(x, q) \vee \nu_A(y, q) = 0\vee 0 = 0$. Therefore, $\nu_A(xy, q) = 0$. We get xy in H. Therefore, H is a subsemigroup of R. Hence H is either empty or is a subsemigroup of R.

2.2 Theorem: Let A be a Q-intuitionistic L-fuzzy subsemigroup of a semigroup (R, ·). (i) If $\mu_A(xy, q)=0$, then either $\mu_A(x, q)=0$ or $\mu_A(y, q) = 0$, for all x and y in R and q in Q.
(ii) If $\nu_A(xy, q)=1$, then either $\nu_A(x, q)=1$ or $\nu_A(y, q) = 1$, for all x and y in R and q in Q.
Proof: Let x and y in R and q in Q. (i) By the definition $\mu_A(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$ which implies that $0 \geq \mu_A(x, q) \wedge \mu_A(y, q)$. Therefore, either $\mu_A(x, q) = 0$ or $\mu_A(y, q) = 0$.
(ii) By the definition $\nu_A(xy, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$ which implies that $1 \leq \nu_A(x, q) \vee \nu_A(y, q)$. Therefore, either $\nu_A(x, q) = 1$ or $\nu_A(y, q) = 1$.

2.3 Theorem: If A is a Q-intuitionistic L-fuzzy subsemigroup of a semigroup (R, ·), then $\Box$A is a Q-intuitionistic L-fuzzy subsemigroup of R.
Proof: Let A be a Q-intuitionistic L-fuzzy subsemigroup of a semigroup R. Consider A = { $\langle$ (x, q), $\mu_A(x, q)$, $\nu_A(x, q)$ $\rangle$ }, for all x in R and q in Q, we take $\Box$A=B={$\langle$ (x, q), $\mu_B(x, q)$, $\nu_B(x, q)$ $\rangle$ }, where $\mu_B(x, q) = \mu_A(x, q)$, $\nu_B(x, q) = 1- \mu_A(x, q)$. Clearly, $\mu_B(xy, q) \geq \mu_B(x, q) \wedge \mu_B(y, q)$, for all x and y in R and q in Q. Since A is a Q-intuitionistic L-fuzzy subsemigroup of R, we have $\mu_A(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$ }, for all x and y in R and q in Q, which implies that $1- \nu_B(xy, q) \geq (1- \nu_B(x, q)) \wedge (1- \nu_B(y, q))$ which implies that $\nu_B(xy, q) \leq 1-\{ (1- \nu_B(x, q)) \wedge (1- \nu_B(y, q)) \}= \nu_B(x, q) \vee \nu_B(y, q)$. Therefore, $\nu_B(xy, q) \leq \nu_B(x, q) \vee \nu_B(y, q)$, for all x and y in R and q in Q. Hence B = $\Box$A is a Q-intuitionistic L-fuzzy subsemigroup of a semigroup R.
Remark: The converse of the above theorem is not true. It is shown by the following example:

Consider the semigroup Z_5 = { 0, 1, 2, 3, 4 } with addition modulo 5 operation and Q = {q}. Then A ={ $\langle$ (0, q), 0.7, 0.2 $\rangle$, $\langle$(1, q), 0.5, 0.1$\rangle$, $\langle$ (2, q), 0.5, 0.4$\rangle$, $\langle$ (3, q), 0.5, 0.1$\rangle$, $\langle$ (4, q), 0.5, 0.1$\rangle$ } is not a Q-intuitionistic L-fuzzy subsemigroup of Z_5, but $\Box$A ={ $\langle$ (0, q), 0.7, 0.3 $\rangle$, $\langle$(1, q), 0.5, 0.5$\rangle$, $\langle$ (2, q), 0.5, 0.5$\rangle$, $\langle$ (3, q), 0.5, 0.5$\rangle$, $\langle$(4, q), 0.5, 0.5$\rangle$ } is a Q-intuitionistic L-fuzzy subsemigroup of Z_5.

2.4 Theorem: If A is a Q-intuitionistic L-fuzzy subsemigroup of a semigroup (R, ·), then ◊A is a Q-intuitionistic L-fuzzy subsemigroup of R.
Proof: Let A be a Q-intuitionistic L-fuzzy subsemigroup of a semigroup R. That is A= {$\langle$ (x, q), $\mu_A(x, q)$, $\nu_A(x, q)$ $\rangle$}, for all x in R and q in Q. Let ◊A = B={$\langle$ (x, q), $\mu_B(x, q)$, $\nu_B(x, q)$ $\rangle$ }, where $\mu_B(x, q) = 1-\nu_A(x, q)$, $\nu_B(x, q) = \nu_A(x, q)$. Clearly, $\nu_B(xy, q) \leq \nu_B(x, q) \vee \nu_B(y, q)$, for all x and y in R and q in Q. Since A is a Q-intuitionistic L-fuzzy subsemigroup of R, we have $\nu_A(xy, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$, for all x and y in R and q in Q, which implies that $1-\mu_B(xy, q) \leq \{ (1-\mu_B(x, q)) \vee (1-\mu_B(y, q)) \}$, which implies that $\mu_B(xy, q) \geq 1-\{ (1-\mu_B(x, q)) \vee (1-\mu_B(y, q)) \}= \mu_B(x, q) \wedge \mu_B(y, q)$. Therefore, $\mu_B(xy, q) \geq \mu_B(x, q) \wedge \mu_B(y, q)$, for all x and y in R and q in Q. Hence B = ◊A is a Q-intuitionistic L-fuzzy subsemigroup of a semigroup R.
Remark: The converse of the above theorem is not true. It is shown by the following example:

Consider the semigroup Z_5 = { 0, 1, 2, 3, 4 } with addition modulo 5 operation and Q = { q }. Then A ={ $\langle$ (0, q), 0.5, 0.1 $\rangle$, $\langle$ (1, q), 0.6, 0.4$\rangle$, $\langle$ (2, q), 0.5, 0.4$\rangle$, $\langle$ (3, q), 0.6, 0.4$\rangle$, $\langle$(4, q), 0.6, 0.4$\rangle$ } is not a Q-intuitionistic L-fuzzy subsemigroup of Z_5, but ◊A = ={ $\langle$ (0, q), 0.9, 0.1 $\rangle$, $\langle$ (1, q), 0.6, 0.4$\rangle$, $\langle$ (2, q), 0.6, 0.4$\rangle$, $\langle$ (3, q), 0.6, 0.4$\rangle$, $\langle$ (4, q), 0.6, 0.4$\rangle$ } is a Q-intuitionistic L-fuzzy subsemigroup of Z_5.

2.5 Theorem: Let A be a Q-intuitionistic L-fuzzy subsemigroup of a semigroup H and f is an isomorphism from a semigroup R onto H. Then A◦f is a Q-intuitionistic L-fuzzy subsemigroup of R.
***Proof*:** Let x and y in R and A be a Q-intuitionistic L-fuzzy subsemigroup of the semigroup H and Q be a non-empty set. Then we have, $(\mu_A \circ f)(xy, q) = \mu_A(f(xy), q) = \mu_A (f(x)f(y), q) \geq \mu_A(f(x), q)\wedge \mu_A(f(y), q) \geq (\mu_A \circ f)(x, q)\wedge(\mu_A \circ f)(y, q)$, which implies that $(\mu_A \circ f)(xy, q) \geq (\mu_A \circ f)(x, q)\wedge(\mu_A \circ f) (y, q)$, for all x and y in R and q in Q. Then we have, $(\nu_A \circ f)(xy, q) = \nu_A(f(xy), q) =\nu_A(f(x)f(y), q) \leq \nu_A(f(x), q) \vee \nu_A(f(y), q) \leq (\nu_A \circ f)(x, q) \vee (\nu_A \circ f)(y, q)$, which implies that $(\nu_A \circ f)(xy, q) \leq (\nu_A \circ f) (x, q) \vee (\nu_A \circ f)(y, q)$, for all x and y in R and q in Q. Therefore (A◦f) is a Q-intuitionistic L-fuzzy subsemigroup of a semigroup R.

2.6 Theorem: Let A be a Q-intuitionistic L-fuzzy subsemigroup of a semigroup H and f is an anti-isomorphism from a semigroup R onto H. Then A◦f is a Q-intuitionistic L-fuzzy subsemigroup of R.
***Proof*:** Let x and y in R and A be a Q-intuitionistic L-fuzzy subsemigroup of the semigroup H and Q be a non-empty set. Then we have, $(\mu_A \circ f)(xy, q) = \mu_A(f(xy), q) = \mu_A(f(y)f(x), q) \geq \mu_A(f(x), q)\wedge \mu_A(f(y), q) \geq (\mu_A \circ f)(x, q)\wedge(\mu_A \circ f)(y, q)$, which implies that $(\mu_A \circ f)(xy, q) \geq (\mu_A \circ f)(x, q) \wedge (\mu_A \circ f) (y, q)$, for all x and y in R and q in Q. Then we have, $(\nu_A \circ f)(xy, q) = \nu_A(f(xy), q) = \nu_A(f(y)f(x), q) \leq \nu_A(f(x), q) \vee \nu_A(f(y), q) \leq (\nu_A \circ f)(x, q) \vee (\nu_A \circ f)(y, q)$, which implies that $(\nu_A \circ f)(xy, q) \leq (\nu_A \circ f)(x, q) \vee (\nu_A \circ f)(y, q)$, for all x and y in R and q in Q. Therefore (A◦f) is a Q-intuitionistic L-fuzzy subsemigroup of a semigroup R.

2.7 Theorem: Let A be a Q-intuitionistic L-fuzzy subsemigroup of a semigroup R, then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is a Q-intuitionistic L-fuzzy subsemigroup of the semigroup R, for every a in R.

Proof: Let A be a Q-intuitionistic L-fuzzy subsemigroup of the semigroup R. For every x and y in R and q in Q, we have, $((a\mu_A)^p)(xy, q) = p(a)\, \mu_A(xy, q) \geq p(a)\{\mu_A(x, q) \wedge \mu_A(y, q)\} = p(a)\, \mu_A(x, q) \wedge p(a)\, \mu_A(y, q) = ((a\mu_A)^p)(x, q) \wedge ((a\mu_A)^p)(y, q)$.Therefore, $((a\mu_A)^p)(xy, q) \geq ((a\mu_A)^p)(x, q) \wedge ((a\mu_A)^p)(y, q)$, for x and y in R and q in Q. And, $((a\nu_A)^p)(xy, q) = p(a)\, \nu_A(xy, q) \leq p(a)\{\nu_A(x, q) \vee \nu_A(y, q)\} = p(a)\, \nu_A(x, q) \vee p(a)\, \nu_A(y, q) = ((a\nu_A)^p)(x, q) \vee ((a\nu_A)^p)(y, q)$. Therefore, $((a\nu_A)^p)(xy, q) \leq ((a\nu_A)^p)(x, q) \vee ((a\nu_A)^p)(y, q)$, for x and y in R and q in Q. Hence $(aA)^p$ is a Q-intuitionistic L-fuzzy subsemigroup of the semigroup R.

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